

On achromatic and pseudoachromatic indices of finite spaces

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A *decomposition* of a simple graph $G = (V(G), E(G))$ is a pair $[G, \mathcal{D}]$ where \mathcal{D} is a set of induced subgraphs of G , such that every edge of G belongs to exactly one subgraph in \mathcal{D} . A *coloring* of a decomposition $[G, \mathcal{D}]$ with k colors is a surjective function that assigns to edges of G a color from a k -set of colors, such that all edges of $H \in \mathcal{D}$ have the same color. A coloring of $[G, \mathcal{D}]$ with k colors is *proper*, if for all $H_1, H_2 \in \mathcal{D}$ with $H_1 \neq H_2$ and $V(H_1) \cap V(H_2) \neq \emptyset$, then $E(H_1)$ and $E(H_2)$ have different colors. The *chromatic index* $\chi'([G, \mathcal{D}])$ of a decomposition is the smallest number k for which there exists a proper coloring of $[G, \mathcal{D}]$ with k colors. A coloring of $[G, \mathcal{D}]$ with k colors is *complete* if each pair of colors appears on at least a vertex of G . The *pseudoachromatic index* $\psi'([G, \mathcal{D}])$ of a decomposition is the largest number k for which there exist a complete coloring with k colors. The *achromatic index* $\alpha'([G, \mathcal{D}])$ of a decomposition is the largest number k for which there exist a proper and complete coloring with k colors. If $\mathcal{D} = E(G)$ then $\chi'([G, E])$, $\alpha'([G, E])$ and $\psi'([G, E])$ are the usual *chromatic*, *achromatic* and *pseudoachromatic indices* of G , respectively.

Clearly we have that

$$\chi'([G, \mathcal{D}]) \leq \alpha'([G, \mathcal{D}]) \leq \psi'([G, \mathcal{D}]).$$

Designs define decompositions of the corresponding complete graphs in the natural way. Identify the points of a (v, κ) -design D with the set of vertices of the complete graph K_v . Then the set of points of each block of D induces in K_v a subgraph isomorphic to K_κ and these subgraphs give a decomposition of K_v .

In this talk we consider some decompositions of the complete graphs coming from the line-sets of finite affine and projective spaces $\text{AG}(n, q)$ and $\text{PG}(n, q)$. We determine the exact values of the achromatic and pseudoachromatic indices in the planar cases and give estimates on the indices if $n > 2$.