On achromatic and pseudoachromatic indices of affine spaces

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joint work with G. Araujo-Pardo, C. Rubio-Montiel and A. Vázquez-Ávila

A *decomposition* of a simple graph $G = (V(G), E(G))$ is a pair $[G, \mathcal{D}]$ where $\mathcal{D}$ is a set of induced subgraphs of G, such that every edge of G belongs to exactly one subgraph in $\mathcal{D}$. A *coloring* of a decomposition $[G, \mathcal{D}]$ with k colors is a surjective function that assigns to edges of G a color from a k-set of colors, such that all edges of $H \in \mathcal{D}$ have the same color. A coloring of $[G, \mathcal{D}]$ with k colors is *proper*, if for all $H_1, H_2 \in \mathcal{D}$ with $H_1 \neq H_2$ and $V(H_1) \cap V(H_2) \neq \emptyset$, then $E(H_1)$ and $E(H_2)$ have different colors. The *chromatic index* $\chi'([G, \mathcal{D})]$ of a decomposition is the smallest number k for which there exists a proper coloring of $[G, \mathcal{D}]$ with k colors. A coloring of $[G, \mathcal{D}]$ with k colors is *complete* if each pair of colors appears on at least a vertex of G. The *pseudoachromatic index* $\psi'([G, \mathcal{D}])$ of a decomposition is the largest number k for which there exist a complete coloring with k colors. The *achromatic index* $\alpha'([G, \mathcal{D}])$ of a decomposition is the largest number k for which there exist a proper and complete coloring with k colors. If $\mathcal{D} = E(G)$ then $\chi'([G, E])$, $\alpha'([G, E])$ and $\psi'([G, E])$ are the usual *chromatic*, *achromatic* and *pseudoachromatic indices* of G, respectively.

Clearly we have that

$$\chi'([G, \mathcal{D}]) \leq \alpha'([G, \mathcal{D}]) \leq \psi'([G, \mathcal{D}]).$$

Designs define decompositions of the corresponding complete graphs in the natural way. Identify the points of a (v, κ)-design $D = (\mathcal{V}, \mathcal{B})$ with the set of vertices of the complete graph K_v. Then the set of points of each block of D induces in K_v a subgraph isomorphic to K_κ and these subgraphs give a decomposition of K_v.

In this talk we consider the decomposition of K_{q^n} coming from the line-set $\mathcal{L}$ of the finite affine space $\mathrm{AG}(n, q)$. We prove that $\psi'([K_{q^2}, \mathcal{L}]) = \lfloor \frac{(q+1)^2}{2} \rfloor$ and we give give estimates on achromatic and pseudoachromatic indices of $[K_{q^n}, \mathcal{L}]$ for $n > 2$.