Aart Blokhuis: Flat-containing and shift-blocking sets

For a pair of non-negative integers $r \geq d$, how small can a subset $G \subset V = V(r,2) = GF(2)^r$ be, given that for any vector $v \in V$, there is a d-flat passing through v and contained in $G \cup \{v\}$?
Equivalently, but looking at the complement: how large can a subset $B \subset V$ be, given that for any $v \in V$ there is a linear d-subspace not blocked non-trivially by the translate $B + v$?
We prove a number of lower and upper bounds.