Abstract: I will go into some detail into the proof of the main ingredients of a recent important result on planar arcs by Ball and Lavrauw:
Assume A is an arc in $PG(2,q)$ that is not contained in a conic.
If A *is not* contained in a curve of deg t, then it is contained in the intersection of two curves of deg $\leq t + p^{\lfloor \log_p t \rfloor}$ not sharing a common component.
If A *is* contained in a curve of deg t and $p^{\lfloor \log_p t \rfloor}(t+\frac{1}{2}p^{\lfloor \log_p t \rfloor}+\frac{3}{2}) \leq \frac{1}{2}(t+2)(t+1)$ then there is another curve of deg at most $t + p^{\lfloor \log_p t \rfloor}$ containing A and sharing no common component with ϕ.
Important consequences of this are:
A is contained in a conic if
(i) q is an odd square, and $|A| \geq q - \sqrt{q} + 3 + \sqrt{q}/p$, or
(ii) q prime, $|A| \geq q - \sqrt{q} + \frac{7}{2}$.